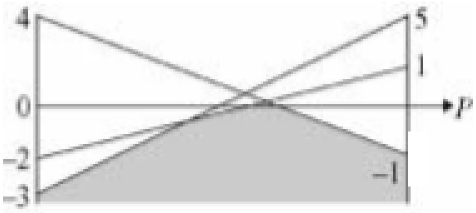
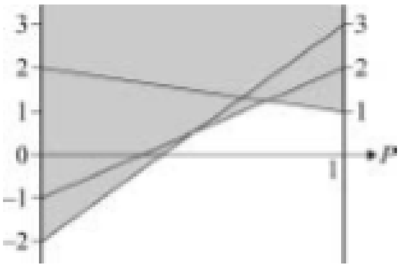
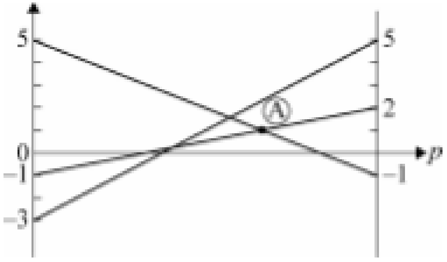
Decision 2 Game Theory Answers

6 (a)	$(-2,2,4)<(2,4,5)$ So S_1 dominated by S_2	E1		
	$\begin{pmatrix}4\\5\\2\end{pmatrix}>\begin{pmatrix}2\\4\\1\end{pmatrix}$			note > sign
	So C_3 dominated by C_2	E1	2	
(b)	2×2 game now $\begin{array}{c\|cc} & c_1 & c_2 \\ \hline s_2 & 2 & 4 \\ s_3 & 5 & 1 \end{array}$			
	Minimum of rows $(2,4)=2$ Minimum of $(5,1)=1$	M1		correct method for either S or C
	Choose maximum = $\textcircled{2}$	A1		play safe for Sam is S_2
	Max of column 1 = max (2,5) = 5 Max of column 2= max (4,1) = 4 Choose minimum = 4	A1		play safe for computer is C_2
	Since $2 \neq 4 \Rightarrow$ not stable solution	E1	4	
(c)(i)	Computer picks C_1 Expected game $=2p+5(1-p)$	M1		
	$=5-3p$	A1		
	Computer picks C_2 Expected gain $=4p+(1-p)$ $=1+3p$	A1	3	
(ii)	Best mixed strategy $5-3p=1+3p$	M1		
	$\Rightarrow p=\frac{2}{3}$	A1	2	
(iii)	Expected points gain $=5-3\times\left(\frac{2}{3}\right)$			Or $1+3\left(\frac{2}{3}\right)$
	$=3$	B1	1	
	Total		**12**	

6(a)	Gain for Rowan +gain for Colleen in each strategy = 0	E1	1	Gain for one = loss of other
(b)	$\begin{array}{lrrr\|l} & & & & \underline{\text{min}} \\ & -3 & -4 & 1 & -4 \\ & 1 & 5 & -1 & \boxed{-1} \\ & -2 & -3 & 4 & -3 \\ \hline \text{Max} & \boxed{1} & 5 & 4 & \end{array}$	M1		minimum of rows & max of columns or maximum of minima or minimax
		A1		All values correct (seen) or words maximin and minimax highlighted
	$1 \neq -1 \Rightarrow$ no stable solution	E1	3	
(c)	R_3 dominates R_1 $(-3,-4,1) < (-2,-3,4)$ so never play R_1	E1	1	
(d)(i)	R chooses R_2 with prob p	M1		Attempt at one expression
	$\Rightarrow$ choose R_3 with prob $1-p$ $\Rightarrow$ expected gain when C plays $C_1: p-2(1-p)=3p-2$ $C_2: 5p-3(1-p)=8p-3$ $C_3: -p+4(1-p)=4-5p$	A1		All correct unsimplified
	Plot expected gains for $0 \le p \le 1$	M1		
		A1		Condone mirror image
	Choosing their "highest" point C_1 & C_3 intersect $\Rightarrow 3p-2=4-5p$	M1		Any 2 lines
	$\Rightarrow p=\frac{3}{4}$	A1		
	$\left.\begin{array}{l}\Rightarrow \text{play } R_2 \text{ with prob } \frac{3}{4} \\ \text{and } R_3 \text{ with prob } \frac{1}{4}\end{array}\right\}$	E1√	7	Statement of strategy
(ii)	Value of game is $3 \times \frac{3}{4} - 2 = \frac{1}{4}$	B1	1	CSO or equivalent, eg 0.25
	Total		**13**	

4(a)(i)	Row min − 4 − 2 − 1	M1		Attempt at row minimum and column maximum
	Col max 5 − 1 3	A1		all figures correct
	min (col max) = max (row min) ⇒ stable solution	E1	3	
(ii)	Ros plays III and Col plays Y	B1		
	value of game = −1	B1	2	
(b)(i)	Ros plays R_1 with probability p and R_2 with probability $1 - p$			
	Expected gains when Col plays:			
	$C_1 : 3p - 2(1 - p) = 5p - 2$			
	$C_2 : 2p - (1 - p) = 3p - 1$	M1		attempt at least 2
	$C_3 : p + 2(1 - p) = 2 - p$	A1		correct unsimplified
	Plot expected gains against p for $0 \le p \le 1$	M1		
		A1		correct (must see 0 or 1 on P axis, or implied by their numbers) A0 if not possible to see highest point of region being correct
	Choose highest point of region below lines $\Rightarrow 3p - 1 = 2 - p$	M1		must be this pair of lines or their highest point
	leading to $p = \frac{3}{4}$	A1		
	Therefore Ros plays R_1 with prob $\frac{3}{4}$			
	and plays R_2 with prob $\frac{1}{4}$	B1√	7	ft their p from any lines
(ii)	Value of game = $3 \times \frac{3}{4} - 1$			
	or $\left(2 - \frac{3}{4}\right) = 1\frac{1}{4}$	B1	1	
	Total		**13**	

3(a)(i)	Min $R_1\,(5, 2, -1) = -1$ Min $R_2\,(-3, -1, 5) = -3$ Min $R_3\,(4, 1, -2) = -2$	E1		
	Max min $= -1$ $\Rightarrow$ Play safe strategy R_1	B1	2	
(ii)	Max $C_1 = 5$; max $C_2 = 2$; max $C_3 = 5$ Min $(5, 2, 5) = 2$	M1		
	$2 \neq -1 \Rightarrow$ no stable solution	A1	2	
(b)	$R_3\,(4, 1, -2) < R_1\,(5, 2, -1)$	E1	1	
(c)(i)	C_1 played, expected gain for Rose: $5p + -3(1-p)$	M1		Any correct expected gain unsimplified
	$= 8p - 3$	A1		One correct simplified
	$C_2 : 2p - (1-p) = 3p - 1$ $C_3 : -p + 5(1-p) = 5 - 6p$	A1	3	All correct simplified
(ii)	Expected gain 	M1		Plotting at least 2 lines
		A1	2	All correct with values at $p = 0$ and $p = 1$ indicated
(iii)	Choosing A – highest point in feasible region $\Rightarrow 3p - 1 = 5 - 6p$	M1		Solving this equation
	$9p = 6$ $\Rightarrow p = \frac{2}{3}$	A1		CSO
	$\Rightarrow$ Rose plays R_1 $\frac{2}{3}$ of time and R_2 $\frac{1}{3}$ of time	E1√	3	
(iv)	Value of game $= 3 \times \frac{2}{3} - 1 = 1$	B1	1	Or $5 - 4 = 1$
	Total		**14**	